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Self-similar expansion of laser plasmas with nonlocal heat flux

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A previous hydrodynamic model of the expansion of a laser-produced plasma, using classical (Spitzer) heat flux, is reconsidered with a nonlocal heat flux model. The nonlocal law is shown to be valid beyond the range of validity of the classical law, breaking down ultimately, however, in agreement with recent predictions.

1. Introduction

The expansion flow of plasmas produced by irradiating solid targets with laser light changes nontrivially as one moves from long, low-intensity pulses to short, intense ones. This change has been studied in detail in the limit of large focal spots, when the plasma corona is quasiplanar (Barrero & Sanmartín 1980; Ramis & Sanmartín 1983). A self-similar analysis is then feasible for a simplified model that uses classical Fourier–Spitzer or flux-limited heat transport, a pulse rising linearly in time, and absorption of energy near the critical density (n_c) (see comment in Section 5).

If the pulse is progressively made steeper, the classical analysis predicts a transition between two markedly different regimes at a definite value of the ratio I_o/τ, where I_o and τ are laser peak intensity and pulse duration, respectively. Below transition, the plasma has a finite length at all times and collisions are dominant. Just above transition, however, the plasma extends to infinity at any given time, and the temperature T is finite at vanishing density, the classical, collisional model thus breaking down in the faraway plasma (electron mean free path $\propto T^2/n \to \infty$). Well above transition, the breakdown affects the entire plasma.

Mean free paths may also be comparable to, or larger than, the temperature scale length in the plasma lying between the critical and ablation surfaces, at the overdense end of the corona; heat conduction under such conditions has been a subject of recent interest. A crude approach often used in these studies is to limit the heat flux to a fraction of its free-streaming value using a saturation factor f ($q_{\max} = fn_eT_e^{3/2}/m_e$). A broad range of f values have been proposed ($f = 0.03$–0.7) for a variety of problems (Manheimer & Klein 1975; Max *et al.* 1980; Matte & Virmont 1982; Shkarofsky 1983; Garban-Labaune *et al.* 1985). There also have been attempts to derive local flux laws for conditions less restrictive than local equilibrium (Moses & Duderstadt 1977; Clause & Balescu 1982; Byung 1985). Nonlocal integral flux laws also have been suggested in the last few years (Luciani *et al.* 1983, 1985; Albritton 1983; Albritton *et al.* 1986; Lindman & Swartz 1986; Bendib *et al.* 1988; Minotti & Ferro 1990; Sanmartín *et al.* 1990, 1992; Murtaza *et al.* 1991).

Flux factors and local laws have failed to provide a general recipe for the breakdown of the classical analysis. Nonlocal laws appear more promising and are being used for specific problems in fluid codes, although difficulties have been found when implemented in full numerical computations or extreme conditions (Prasad & Kersaw 1989; Epperlein &

Short 1991). In this article, we show that one can solve the deficiencies of the classical flux in the underdense plasma for a limited range of conditions above transition by using the full formulation by Albritton *et al.* (1986).

These authors considered a weakly collisional plasma at large ion charge number ($Z_i \gg 1$), the mean free path for the high-energy ($\simeq 6.5\ T$), heat-carrying electrons being comparable to the temperature scale length. They obtained an integral expression for the heat flux:

$$q(x) = \frac{-(\lambda_{90}/\lambda_\varepsilon)^{1/2}}{4\pi(3m_e)^{1/2}} \int dx' n' T'^{1/2} \left[\frac{\partial T'}{\partial x'} K(\theta) - \frac{\partial e\phi'_{\rm nl}}{\partial x'} L(\theta) \right], \quad (1)$$

involving a "nonlocal" electric potential $e\phi_{\rm nl}$, which is determined by a second integral equation that expresses the condition of zero electric current:

$$0 = \int dx' n' T'^{-1/2} \left[\frac{\partial T'}{\partial x'} I(\theta) - \frac{\partial e\phi'_{\rm nl}}{\partial x'} J(\theta) \right]. \quad (2)$$

In equations (1) and (2), the propagators I, J, K, and L are universal functions dependent on a normalized distance θ from x' to x; and x, λ_{90}, and λ_ε are space coordinate, electron–ion, and electron–electron mean free paths, respectively. Simpler results for high and low Z_i have been presented by Sanmartín *et al.* (1990, 1992).

It is worth stressing that the nonlocal formulation, which provides a consistent although mathematically complex procedure for evaluating the heat flux when collisions are nonlocal, is compatible with a self-similar solution. In the following sections, we reconsider the self-similar plasma expansion with the above heat flux law and compare results from both the classical and nonlocal laws. In Section 2, we present the equations of the problem. In Section 3, we discuss possible asymptotic behaviors near the plasma–vacuum interphase. Section 4 gives numerical results, and Section 5 summarizes the conclusions.

2. Hydrodynamic equations with integral heat flux

Assuming that the solid target fills the $x < 0$ half-space and that laser light is incident from $x = +\infty$, Barrero and Sanmartín (1980) obtained continuity, momentum, and electron-entropy self-similar equations for the case of a pulse linear in time, $I(t) = I_0 t/\tau$. With appropriate normalization, they obtained [equations (4)–(6) of their article]

$$\frac{d\nu}{d\eta} = \frac{\nu}{\eta - y}\frac{dy}{d\eta} \quad (3)$$

$$y - 4(\eta - y)\frac{dy}{d\eta} = -\frac{4}{\nu}\frac{d(\nu z)}{d\eta} \quad (4)$$

$$\nu\left[z\left(1 + \frac{4}{3}\frac{d\nu}{d\eta}\right) - 2(\eta - y)\frac{dz}{d\eta}\right] = -\frac{d\Omega}{d\eta} + 8\nu_c I\delta(\eta - \eta_c) \quad (5)$$

where

$$\nu = n/n_r, \qquad y = v[(t/\tau)^{1/3}(Z_i kT_r/m_i)^{1/2}]^{-1},$$
$$z = T[(t/\tau)^{2/3}T_r]^{-1}, \qquad \Omega = q[n_r T_r (Z_i T_r/m_i)^{1/2}\tfrac{3}{4}]^{-1},$$
$$\eta = x[(3\tau/4)(t/\tau)^{4/3}(Z_i kT_r/m_i)^{1/2}]^{-1}$$

are self-similar, dimensionless density, fluid velocity, electron temperature, heat flux, and space coordinate, respectively. The temperature reference is $T_r = [(\tfrac{9}{16})(Z_i k^2 \tau n_r/m_i\bar{K})]^{2/3}$, where the reference density n_r is chosen to simplify boundary conditions (Barrero & Sanmartín 1980). The δ term represents energy deposition at the critical density n_c, with $I = (4m_i/9Z_i)^{3/2} I_0 \bar{K}/\tau n_c^2$ a normalized absorbed intensity.

For the classical case, we would have $q = -\bar{K}T^{5/2}\mathrm{d}T/\mathrm{d}x$, or $\Omega = -z^{5/2}\mathrm{d}z/\mathrm{d}\eta$. Here, we use the integral flux law (1) and (2), which, as already mentioned, takes a self-similar form:

$$\Omega(\eta) = \frac{-C}{2.192\pi^{1/2}} \int \mathrm{d}\eta' \nu' z'^{1/2} \left[\frac{\partial z'}{\partial \eta'} K(\theta) - \frac{\partial e\Phi'_{\mathrm{nl}}}{\partial \eta'} L(\theta) \right] \tag{6}$$

$$0 = \int \mathrm{d}\eta' \nu' z'^{-1/2} \left[\frac{\partial z'}{\partial \eta'} I(\theta) - \frac{\partial e\Phi'_{\mathrm{nl}}}{\partial \eta'} J(\theta) \right], \tag{7}$$

where, for instance, ν' is $\nu(\eta')$. We have normalized $e\phi_{\mathrm{nl}}$ with the reference temperature. The nonlocal propagators are

$$P(\theta,\alpha,\beta) = \theta^{2+2\beta} \int_0^\infty \mathrm{d}y \int_0^1 \mathrm{d}y' \frac{y'^\alpha y^\beta}{(1-y')^{1/2}} \exp\left[-\theta^{1/2} y^{1/4} - \frac{1}{y(1-y')} \right], \tag{8}$$

with $I = P(\theta,0,0)$, $J = P(\theta,0,-\frac{1}{4})$, $K = P(\theta,\frac{1}{4},\frac{1}{4})$, and $L = P(\theta,\frac{1}{4},0)$. The θ variable here becomes $\theta(\eta,\eta') = C \left| \int_\eta^{\eta'} \nu'' \mathrm{d}\eta'' \right| / z'^2$, where the constant $C = 128(3\pi)^{-1/2}(m_i/Z_i m_e)^{1/2} \times (1 + Z_i \Lambda_{ei}/\Lambda_{ee})^{-1/2} \gg 1$ has been taken equal to 791 for numerical purposes (m_i/Z_i twice the proton mass and $Z_i\Lambda_{ei}/\Lambda_{ee} = 9$). Note that the temperature $z(\eta')$ was used to define θ, resulting in an extremely large value for C; if an energy of interest, about 6.5 z', was used instead, C would become $791/(6.5)^2 \simeq 18.7$.

For a collision-dominated plasma, the boundary conditions require $y = z = 0$, $\nu z = 1$ at the solid ($\eta = 0$) and $\nu \to 0$, $\Omega \to 0$ at a plasma–vacuum interphase lying at either finite or infinite distance ($y - \eta \to 0$); these conditions suffice to determine the fourth-order system for ν, y, Ω, and z and the reference density n_r. For the nonlocal regime, when equations (6) and (7) are used instead of $\Omega = -z^{5/2}\mathrm{d}z/\mathrm{d}\eta$, one drops the condition on Ω.

Rewriting (6) and (7) as integrals over θ, the combined behavior of the nonlocal propagators $P(\theta)$ (falling off exponentially with θ or $|x' - x|$) and the remaining dominant factor in the integrands ($T^{5/2}\mathrm{d}T/\mathrm{d}x$) gives the flux, when T is steep enough, as the shaded area of figure 1. The heat flux is then delocalized. The characteristic length (L_F in the fig-

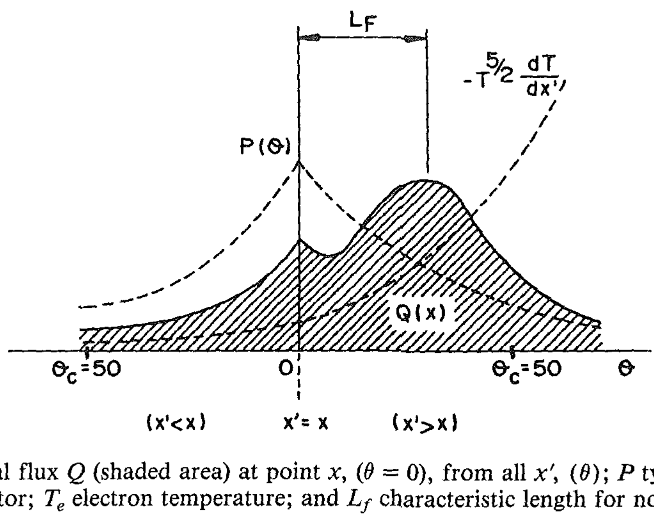

FIGURE 1. Integral flux Q (shaded area) at point x, ($\theta = 0$), from all x', (θ); P typical Albritton's nonlocal propagator; T_e electron temperature; and L_f characteristic length for nonlocal heat flux.

ure), which increases as plasma density decreases, gives the distance from point x, where the heat flux is evaluated, to the plasma location x' contributing most to the heat flux (maximum integrand). In the opposite limit, when the temperature scale length L_T is much larger than L_F, equations (6) and (7) yield $\Omega = -z^{5/2}\mathrm{d}z/\mathrm{d}\eta$ and $\mathrm{d}e\Phi_{\mathrm{nl}}/\mathrm{d}\eta = 5\mathrm{d}z/\mathrm{d}\eta$, showing that the complex integral formulation reduces to the much simpler classical law.

In the most general case, the complete solution to the fluid problem appears to be integrating numerically the system of equations (3)–(7) toward the critical surface both from the solid target and the plasma–vacuum interphase, with appropriate free constants; matching of density, velocity, and temperature at that surface allows one to determine all those constants; the discontinuity in the heat flux at that matching point yields the laser energy deposition. Integration from the interphase is made easier by the introduction of variables in a certain phase space: density $N = \nu/\eta^3$, velocity $Y = (y - \eta)/\eta$, temperature $Z = z/\eta^2$, and heat flux $F = \Omega/\eta^6$. The system of equations (3)–(7) then becomes

$$\frac{\mathrm{d}N}{\mathrm{d}Y} = -\frac{N}{Y}\left[1 + (1 + 4Y)\frac{\mathrm{d}\ln\eta}{\mathrm{d}Y}\right] \tag{9}$$

$$\frac{\mathrm{d}Z}{\mathrm{d}Y} = -\frac{Z}{Y} - Y - \frac{\mathrm{d}\ln\eta}{\mathrm{d}Y}\left[2Z - (1 + Y)\left(\frac{Z}{Y} - 1 - \frac{1}{4}\right)\right] \tag{10}$$

$$\frac{\mathrm{d}F}{\mathrm{d}Y} = -\frac{\mathrm{d}\ln\eta}{\mathrm{d}Y}\left[6F + NZ\left(\frac{7}{3} + \frac{16}{3}Y\right)\right] - \frac{4}{3}NZ - 2NY\frac{\mathrm{d}Z}{\mathrm{d}Y} \tag{11}$$

$$F = \frac{1}{\eta^6}\frac{C}{384\pi^{1/2}}\int \mathrm{d}Y'\left(-\frac{\mathrm{d}\ln\eta'}{\mathrm{d}Y'}\right)\frac{N'F_s'\eta'^6}{Z'^2}\left[K(\theta) - K_e'L(\theta)\right] \tag{12}$$

$$0 = \int \mathrm{d}Y'\left(-\frac{\mathrm{d}\ln\eta'}{\mathrm{d}Y'}\right)\frac{N'F_s'\eta'^4}{Z'^3}\left[I(\theta) - K_e'J(\theta)\right], \tag{13}$$

where $K_e' \equiv (\mathrm{d}e\Phi_{\mathrm{nl}}'/\mathrm{d}\eta')/(\mathrm{d}z'/\mathrm{d}\eta')$ is the nonlocal field temperature gradient ratio and $\theta(Y, Y') = C\left|\int_Y^{Y'} N''\eta''^4(\mathrm{d}\ln\eta''/\mathrm{d}Y'')\mathrm{d}Y''\right|/Z'^2\eta'^4$. Notice that the quantity $F_s = -Z^{5/2}[2Z + (\mathrm{d}Z/\mathrm{d}Y)/(\mathrm{d}\ln\eta/\mathrm{d}Y)]$ would be proportional to the Spitzer heat flux. For the classical regime, setting $F = F_s$ determines $\mathrm{d}\ln\eta/\mathrm{d}Y$ to be used in equations (9)–(11); for the nonlocal regime, equations (12) and (13) determine $\mathrm{d}\ln\eta/\mathrm{d}Y$ (and K_e'). Boundary conditions at the interphase ($Y = 0$) are $N = 0$ and, for dominant collisions there, $F = 0$.

3. Asymptotic solutions near the plasma–vacuum interphase

As often happens when nonlinear diffusion competes with nonlinear conduction (Robert & Soward 1972; Sanz *et al.* 1981), a change of functional behavior (bifurcation) of asymptotic solutions occurs at definite values of an eigenvalue exponent in the solution. Note first that, independently of any specific flux law, equations (9) and (10) for $Y \to 0$ allow two asymptotic laws for the phase variable $Y(\eta)$: (1) $\mathrm{d}\ln\eta/\mathrm{d}Y = -a$, $(a > 0)$; and (2) $\mathrm{d}\ln\eta/\mathrm{d}Y = -b/Y$, $(b > 0)$, for which the interphase lies at finite or infinite distance η_V, respectively. The corresponding laws for phase density and temperature are: (1) $N = N_1 Y^{a-1}$, $(a > 1, N_1 > 0)$, $Z = Y/4 - Y^2(1 - 3a/2)/(a + 1)$; and (2) $\ln N = -b/Y$, $Z = Y/4 + Y^2 3/2$.

Next, using the above behaviors in equation (11) we find, for an interphase lying at infinite distance, (2) $\ln F = -b/Y$. In the opposite case (1), we have $\mathrm{d}F/\mathrm{d}Y = 6aF + N[aZ(7 + 16Y)/3 - 4Z/3 - 2Y\mathrm{d}Z/\mathrm{d}Y]$, and we find three possible situations: (1a) $a = 10/7$, with F, $\mathrm{d}F/\mathrm{d}Y \ll NY$, the behavior of F remaining undetermined; (1b) $a > 10/7$, with $F = Y^{a+1}(N_1/12)(7a - 10)/(a + 1)$; and (1c) $F = F_v(1 + 6aY)$, F_v representing an escaping heat flux at the plasma–vacuum interphase. Barrero and Sanmartín (1980) showed that the classical flux law $F = F_s$ is compatible with behaviors (1a) $a = 10/7$ and (1b) $a = 3/2$, corresponding to their isentropic and transition regimes, respectively; it is also formally compatible with behavior (2), corresponding to their isothermal regime, above transition, when the classical description fails to hold, however. Behavior (1c) and the integral flux law now need further analysis.

Normalizing with the same reference length used for the space coordinate x, the electron–ion mean free path takes the form $\bar{\lambda} = C_1 \eta Z^2/N$, with $C_1 = (9/384)(\pi/2)^{1/2} \times (m_e Z_i/m_i)^{1/2}(1 + Z_i \Lambda_{ei}/\Lambda_{ee}) \ll 1$. Near the vacuum interphase, we have $\bar{\lambda} \simeq C_1 \eta_v / 16 N_1 Y^{a-3}$ for finite η_v and $\ln \bar{\lambda} \simeq k/Y$, k constant, for infinite η_v. This shows that for both the infinite case and the finite case, if $a > 3$, $\bar{\lambda}$ grows indefinitely when approaching the interphase, thus breaking the plasma collisionality. On the contrary, for finite η_v and $a < 3$ we have $\bar{\lambda} \to 0$ as $Y \to 0$ and the plasma remains collisional all the way out to the interphase. Numerical calculations have shown that the dividing case $a = 3$ can be considered collisional because the corresponding value of $\bar{\lambda}$, $C_1 \eta_v / 16 N_1$, is found numerically to be much smaller than unity.

A second point of interest concerns locality. Comparing $\bar{\lambda}$ near the interphase with Y (a measure of phase distance to the vacuum), we find that, for $a < 2$, $\bar{\lambda}/Y$ vanishes as $Y \to 0$, which means that points near Y control the heat flux at Y (localized flux); on the contrary, if $a > 2$, we have $\bar{\lambda}/Y \to \infty$ as $Y \to 0$, that is, the heat flux at Y is controlled by electrons coming from some distance away (delocalized flux).

From a physical point of view, three different regimes appear then in terms of the a parameter, caused by nonlinear competition between diffusion and convection, as already noticed. For $a < 2$, the plasma is collisional and the heat flux is local (classical heat flux); for $2 < a \leq 3$, the plasma is collisional but the heat flow is nonlocal (the classical law fails; the integral law does not); finally, for $a > 3$ the plasma is noncollisional near the interphase (both laws fail).

Applying the above considerations to the integral heat flux law (12) and (13) for $a < 2$, and considering values $|Y' - Y| \ll Y$, allows us to recover classical flux results, $F = Y^{5/2}/128a$ and $K_e = 5$, with just two possible a-values, corresponding to classical regimes at finite interphase distance: isentropic ($a = 10/7$) and transition ($a = 3/2$). For $a > 2$, the $Y' < Y$ (cold) and $Y' > Y$ (hot) regions must be treated separately. The cold region, vanishing with Y, can be neglected against the hot one, which remains finite; then, for $Y = 0$ we obtain asymptotically $K_e = (5a + 7)/(a - 2)$ and a finite, nonzero, heat flux F_v. Finally, use of equations (12) and (13) for a refined discussion of asymptotic behaviors in equation (11) yields the density constant N_1 and the escaping flux F_v as functions of a. We find

$$N_1 = \frac{(4n+3)(4n+33)}{C(2n+3)} \left[\frac{24(4n+33)\Gamma(a_1)}{5(5n+6)\Gamma(a_3)} + \frac{48\Gamma(a_2)}{(n+1)\Gamma(a_4)} \right]^{-1} \tag{14}$$

$$F_v = \frac{4n(4n+3)\Gamma(b_1)\Gamma(b_2)(\Gamma(b_3))^2}{3(2 \times 192\pi^{1/2})(128 \times 3)(16CN_1)^{5n/3}\Gamma(b_4)}, \tag{15}$$

where Γ is the gamma function, $n(a) = 1.5/(a-2)$, $a_1 = (9/4 + 5n/6)$, $a_2 = (1 + n/2)$, $a_3 = (7/4 + 5n/6)$, $a_4 = (1/2 + n/2)$, $b_1 = (6 + 10n/3)$, $b_2 = 5/4$, $b_3 = (1 + 5n/6)$, and $b_4 = (9/2 + 5n/6)$ (figure 2).

The asymptotic behavior for $a > 2$ corresponds to case (1c); for $(2 < a \leq 3)$, it represents a consistent range of solutions with laser energies above the transition, where classical analysis was unsatisfactory. The escaping flux in this range is extremely small ($F < 10^{-7}$) when compared to a typical flux ($\simeq 1$) at the critical surface and can be physically

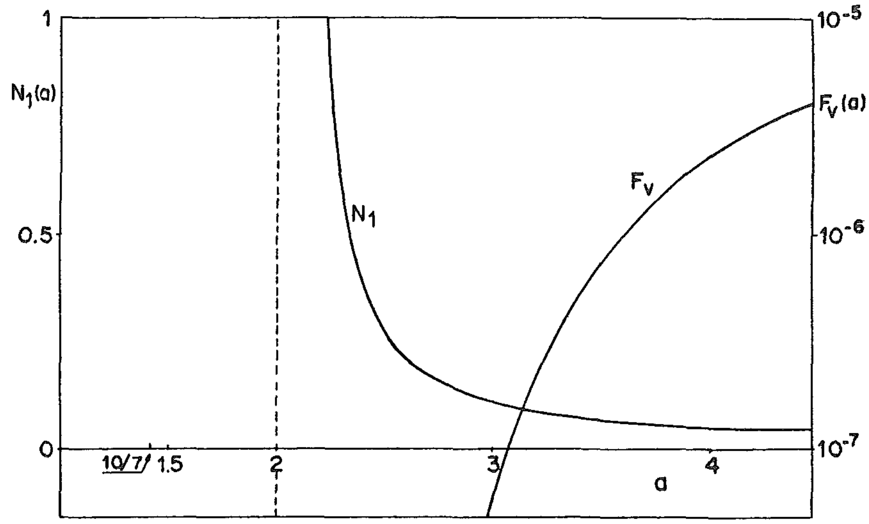

FIGURE 2. Density N_1 and heat flux F_v constants in the nonlocal asymptotic solution (1c).

ignored. As we shall later see, $I \to \infty$ as $a \to \infty$ and then solutions of type (2) appear not to correspond to any real regime.

4. Numerical results above transition

For $2 < a < 3$, the region where nonlocal effects are dominant is small; its phase thickness ΔY can be estimated as the value Y_0 at which the classical phase heat flux F_s becomes equal to the escaping flux F_v of the asymptotic solution: $Y_0 = (128aF_v)^{2/5} \ll 1$.

A first, approximate numerical solution can then be obtained by patching the asymptotic solution: $N = N_1(a)Y^{a-1}$, $Z = Y/4 - Y^2(1 - 3a/2)/(a + 1)$, $\mathrm{d}\ln\eta/\mathrm{d}Y = -a$, and $F = F_v(1 + 6aY)$ for $Y \in [0, Y_0]$ —with a step-by-step integration of equations (9)–(11), with the classical flux law, for $Y > Y_0$. After crossing a first critical sonic point ($Z_s^2 = Y_s$), the solution reaches a second one (the critical surface) where matching to the appropriate solution integrated from the solid gives the complete profile of the phase variables and the energy parameter I as a function of a (figure 3); a further integration of $\eta(Y)$ also allows one to express the plasma thickness η_v as a function of a (figure 3) in a range where the classical analysis showed η_v to be infinite. A plasma of infinite extent appears only in the limit $a, I \to \infty$. As $a \to 2^+$, then, I approaches $I^* \simeq 0.14$ (the transition value) from above and the nonlocal region collapses; for any $I > I^*$ but near to I^*, the transition behavior ($a = 3/2$) is recovered beyond the nonlocal region. Similarly, for any $I < I^*$ but near I^*, that behavior appears beyond a thin isentropic ($a = 10/7$) region that collapses as $I \to I^{*-}$.

A typical solution for the self-similar variables along the plasma, in the regime above transition, with nonlocal effects is shown in figure 4. The final numerical solution of equations (9)–(13) near vacuum requires a global integration of the system. For every value of the a parameter, we establish a fixed interval $0 < Y < Y_M$ [Y_M chosen to recover the classical (local) heat flux at this point], and starting with a zero-order solution for η, N, and Z taken from the previous asymptotic-classical patching we solve the discretized integral equation for K_e' and calculate the first-order solution for F, η, N, and Z following an iter-

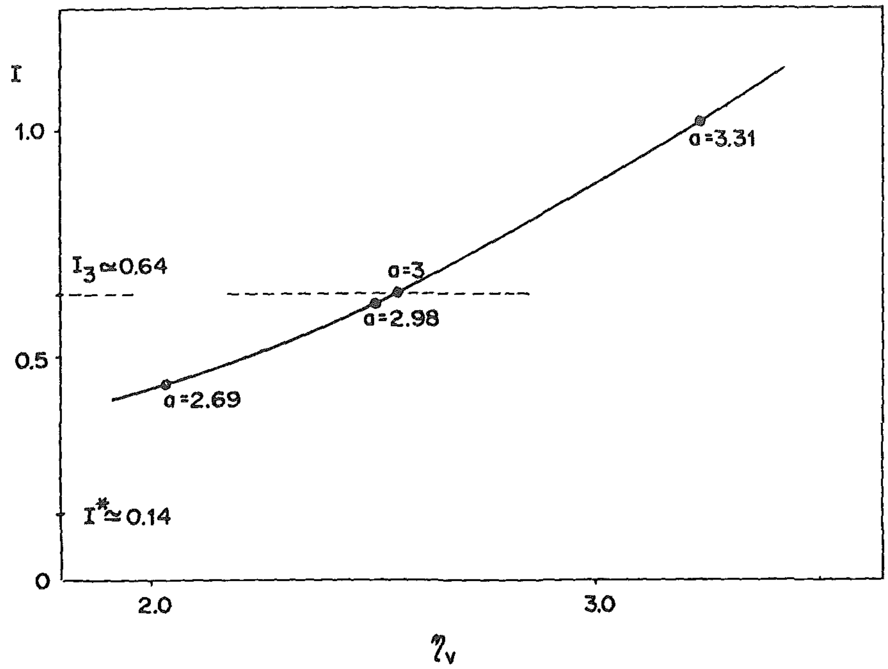

FIGURE 3. Deposition energy I versus plasma thickness η_v and a parameter; I^* transition value.

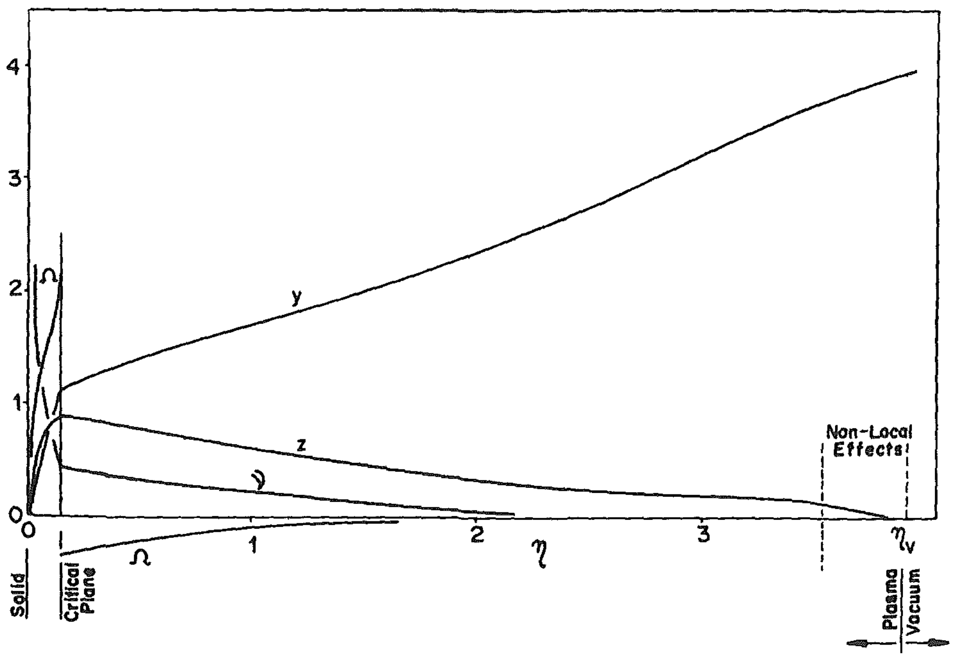

FIGURE 4. Typical profiles of the self-similar variables along the plasma: y velocity, ν density, z temperature, Ω heat flux, η space coordinate.

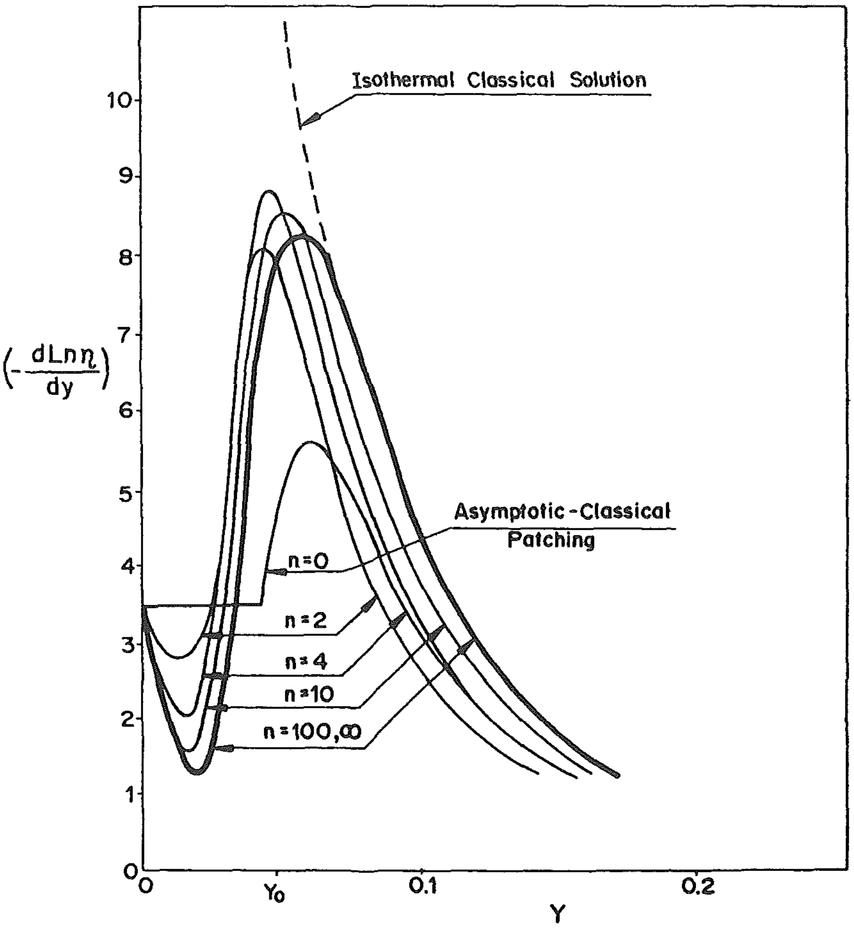

FIGURE 5. Typical iterative solution for $-\mathrm{d}\ln\eta/dY$.

ative procedure that proves to be convergent. Figure 5 shows different stages in the iterative determination of $-\mathrm{d}\ln\eta/\mathrm{d}y$ versus Y for a typical case, and figure 6 compares values for the sonic point (F_s, N_s) from the complete numerical solution and from the patching of asymptotic and classical results; the agreement is good.

5. Summary

We have analyzed the planar expansion of a laser-produced plasma with a nonlocal heat flux formalism. We have shown that such formalism recovers the classical results at and below transition $[I_0\lambda^4/\tau = 1.14\ 10^{11}\ (\mathrm{W/cm^2})\,\mu\mathrm{m}^4/\mathrm{ns}]$ and changes the classical results above it; instead of plasmas of infinite extent, one finds a family of solutions in which the plasma thickness continuously increases with laser energy. The solution at transition is the lower limit of this family as laser intensity goes to the transition value; the infinite plasma solution is the family limit for laser intensity going to infinity.

In a certain intensity range (up to five times the transition value), nonlocal effects are restricted to a small portion of the underdense plasma and the vacuum interphase remains

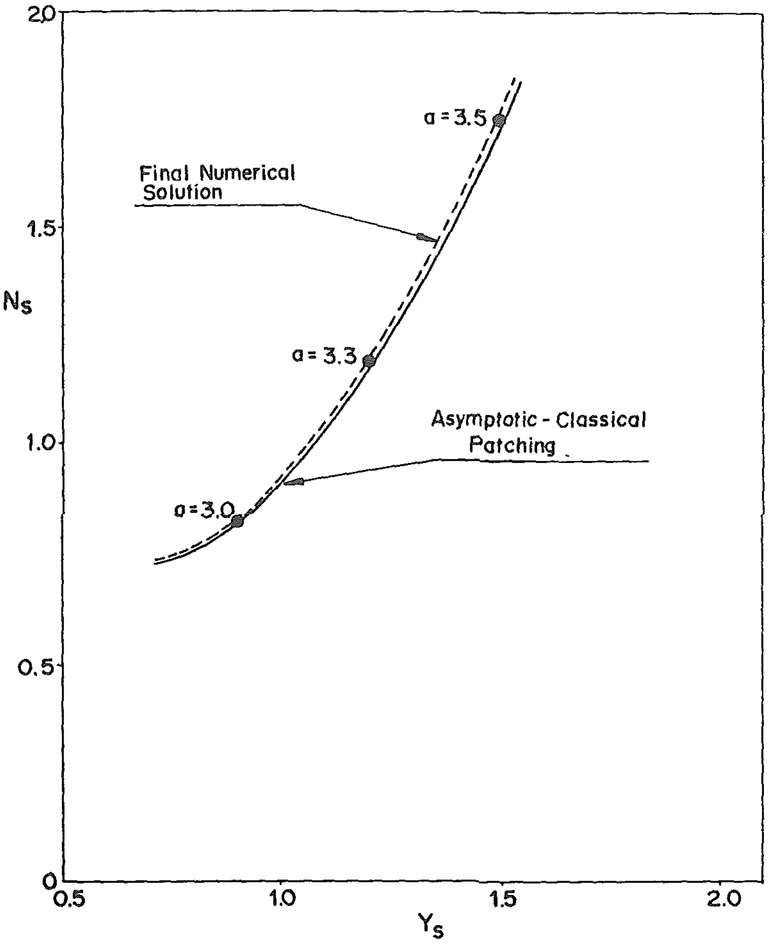

FIGURE 6. Comparison of the final numerical solution with the asymptotic-classical patching.

collisional but nonlocally controlled, letting escape a small (negligible) fraction of energy that barely modifies the classical results on ablation pressure and mass flow rate. Well above this range, the solution would give a nonnegligible escaping flux, the plasma–vacuum interphase would not be collision dominated, and the nonlocal effects would affect the whole plasma.

We recall, finally, that in our model absorption was assumed to occur at the critical surface. However, from the numerical results and using the expressions for ν, z, η, and T_r given previously in equations (3)–(5), we find that the inverse Bremsstrahlung absorption

$$A = \int_{x_c}^{x_v} \frac{n\,\mathrm{d}x}{n_c c \tau_e (1 - n/n_c)^{1/2}}$$

where $\tau_e = m_e \bar{K} T^{3/2}/\gamma_0 n$ ($\gamma_0 \cong 13.6$) is about unity for I around its transition value. The same numerical results show that two-thirds of the absorption occurs in the first 10 or 15% of the underdense plasma length. This makes the model (which simplifies the analysis extraordinarily) a reasonable one.

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